B.Sc. 5th Semester Assignment

Department of Mathematics,

Mugberia Gangadhar Mahavidyalaya

(Group Theory-1)

Paper –C6T

1. In a commutative group (G,*) , Prove that $(a * b)^{-1}$=$a^{-1} * b^{-1}$ for all a, b in G.
2. Let (G,*) be an abelian Group and H = $\{a^2 : a \in G\}$. Prove that H is a subgroup of G.
3. Show that the Set S={$\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} : x \in \mathbb{R}$} is a subgroup of the group of all second order non singular real Matrices.
4. Prove that the set $\mathbb{Q} - \{1\}$ where $\mathbb{Q}$ is the set of all rational number forms a abelian group under the composition * defind by $a * b = a + b - ab$
5. Prove that a group G is abelian if $b^{-1}a^{-1}ba = e$ for all a, b in G.
6. Show that the group $S_3 is$ non abelian
7. Show that the group G={1,ω, ω^2} is cyclic .
8. Let (G,*) be an infinite cyclic group generated by a , Prove that a and a^{-1} are only generators of the group .
9. Prove that the set of all integers which are multiple of 5 is a subgroup of (Z,+).
10. If G be an abelian group , then prove that $(a * b)^n$=$a^n * b^n$
For all integers n.

11. The centre Z of g is define by Z(G)={z$\in G$:zx=xz for all x$\in G$ }.
Prove that Z(G) is a normal subgroup of G.
12. If G be a group in which $(ab)^3 = a^3b^3$ for all a, b in G. Prove that H =$\{x^3 : x \in G\}$ is a normal subgroup of G.
13. If G be a group and H is a subgroup of index 2 in G, Prove that H is a normal subgroup.
14. Prove that every proper subgroup of the symmetric group S_3 is cyclic.
15. Show that the Commutative subgroup of any group is a normal subgroup.

16. Let H and K be two subgroups of (G,*) then HK is a subgroup of G iff HK=KH.
17. A subgroup H of a group G is normal iff xHx^{-1}=H for all $x \in G$.
18. Prove that the set of Matrices of the form $\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$ is a abelian group under matrix multiplication, where θ *is* real.
19. Give an example to show that if His normal subgroup of G and K is normal subgroup of H then k may not be a normal subgroup of G.
20. Show that every subgroup of a cyclic group is normal.
21. Prove that the ring s={$a + b\sqrt{5}: a, b$ *aer integers*} is a commutative group with respect to addition.
22. Show that the set s={$\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}: x$ *are real and* $x \neq 0$} forms a normal subgroup of GL(2,R), the group of all real non singular 2×2 matrices.
23. Show that every subgroup of a abelian group is normal.
24. Show that the set s={$\begin{pmatrix} x & y \\ -y & x \end{pmatrix}: x, y$ *are real and*$x, y \neq 0$} is a group under matrix multiplication.
25. Prove that residue class modulo 6,z_6 is is a commutative group with respect to addition.
26. Show that the set S={$\begin{pmatrix} x & y \\ 0 & x \end{pmatrix}: x, y$ *are real and* $x \neq 0$} is a commutative group under matrix multiplication.
27. Give an example to show that union of two subgroup may not be a subgroup.
28. Prove that the set of all integers of the form 2n where n is any integer is a commutative group with respect to addition.
29. .Using Lagrange's theorem prove that every group of prime order is cyclic.
30. Any two right or left cosets of a subgroup are either disjoint or identical.
31. Prove that every quotient group of a cyclic group is cyclic.
32. Show that two right cosets Ha,Hb are distinct iff the two left cosets $a^{-1}H, b^{-1}H$ are distinct.
33. Let Z be the centre of a group . If $G/_H$ is cyclic, prove that G is abelian.
34. If G is a group and H is a subgroup of index 2 in G , prove that H is a normal subgroup.